AN ENERGY EFFICIENT CDMA SCHEME FOR IN-SITU COMMUNICATIONS ON MARS

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Abstract— **To extend the battery life of a Martian surface unit it is desired to minimize the energy consumed by the surface to orbit communications transceiver. The performance of three different transmission schemes are evaluated in terms of the energy-per-bit required by the transmit power amplifier to achieve a desired energy-per-bit at the receiver. A fixed-power variable-rate transmission scheme is shown to provide the largest savings in energy. Using this as the basis for selecting a multiple access system, a code division multiple access system is presented and its performance is analyzed.**

I. Introduction

A multiple access communication system for surface to orbit communications around Mars must meet a specified performance level while expending a minimum of energy at the surface unit. As the data volume on the uplink exceeds that of the downlink, and the amplification requirements of the transmitters are greater than those of the receiver, the surface unit's transmitter dominates the power consumption of the communications transceiver. The multiple access communication system should therefore be designed to minimize the energy used by the surface unit's transmit power amplifier. This requires looking not only at the power of the transmitted signal, but also at the efficiency of its generation.

System models of both the uplink channel and power amplifier are given in Sections II and III. This is followed by an analysis of the transmit power amplifier energy requirement for fixed-power fixed-rate, variable-power fixed-rate, and variable-rate fixed-power transmission schemes. Using this information, a multiple access scheme which minimizes the energy necessary for a communications uplink on Mars is presented and its performance is analyzed.

II. Channel

In order for a satellite orbiting Mars to cover most of the surface and to appear at roughly the same time each day to a surface unit, it must follow one of a class of low circular polar orbits which are known as helo-synchronous. One such orbit follows a path approximately 400km above the surface of Mars. The number of passes, their duration, as well as the path distance between a surface unit and the satellite can be solved by taking into account both the satellite orbit as well as the rotation of the planet. As a simple example, consider a surface unit located at one of the Martian poles for which we can ignore the rotation of planet. In addition, we consider Mars as a perfect sphere with a radius of $r_u = 3393.5km$, a mass of $M = 6.42 \cdot 10^{23}g$, and possessing a uniform gravitational field.

Using simple geometry and Newtonian mechanics, the time varying distance between the surface unit and the satellite, the elevation angle, ϕ, and the angular velocity of the satellite, ω_p, can be easily solved. The instantaneous received signal power, accounting only for the time varying free-space path loss, can then be written in terms of the power flux density (PFD) and the aperture of the antenna, A_e, [1], as

$$P_R(t) = A_e P_{rf} \cdot G_T \frac{1}{4\pi K^2 (\alpha - \cos(\omega_p t))}, \tag{1}$$

where P_{rf} is the transmitted signal power, G_T is the antenna gain, $\alpha = \frac{r_s^2 + r_u^2}{2 r_u r_s}$, $K = \sqrt{2 r_u r_s}$, and r_s is the distance from the center of mars to the satellite. For hardware complexity reasons, we are only considering omni-directional antennas.

Similarly the duration of the transmission window, T, can be found in terms of the lowest possible elevation angle at which the surface unit can 'see' the satellite, ϕ_{min}; this gives

$$T = \frac{2}{\omega_p} \left(\arccos\left(\frac{r_u}{r_s} \cos(\phi_{min}) \right) - \phi_{min} \right).$$

From the surface unit's perspective, as the satellite moves from the horizon to overhead, the distance between the unit and the satellite will decrease. The rate at which this occurs along with the frequency of the transmission will determine the Doppler shift of the transmitted signal. The wave properties of the transmitted signal will be affected by this velocity, resulting in an extension or contraction of the wavelength over the period of the transmitted signal. Accounting for the time dilation caused by the relative motion of the satellite with the surface unit we can write the Doppler shift, f_d, of the signal about the carrier as

$$f_d(t) = f_c \cdot \left(\sqrt{\frac{1 - \frac{v(t)}{c}}{1 + \frac{v(t)}{c}}} - 1 \right), \tag{2}$$

where $v(t) = 0.5 K \omega_p \sin(\omega_p t) / \sqrt{\alpha - \cos(\omega_p t)}$ is the relative velocity of the satellite toward the surface unit.

III. Power Amplifier

There are several classes of power amplifiers, each having a general range of efficiencies for that class. As the power consumption is a primary concern, the efficiency of the amplifier is paramount. For this reason a constant envelope modulation

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scheme is used; this allows for the use of a reduced conduction angle power amplifier, known as class AB, B, and C power amplifiers. These amplifiers suffer from amplitude modulation (AM) to phase modulation (PM) distortion even when operated below the compression point (linear region)[2] and are therefore restricted to constant envelope modulation schemes. There are numerous variations on amplifier designs which can be made to increase both the amplifier efficiency and/or linearity for the different transmission schemes. The comparisons in this paper will not consider these extensions and will instead use a simple idealized class B amplifier model for all the proposed techniques. This will insure a uniform comparison, as well as conservatively biasing the results since most enhancements would result in greater efficiency.

For a sinusoid, $v_{in}(t) = \gamma\cos(\omega_c t)$, with a magnitude γ, ranging from zero to unity, acting as an input to a class B amplifier the average power consumed by the power supply, P_{pa}, to generate an RF output power of P_{rf} is equal to $P_{rf}\frac{4}{\gamma\pi}$ [2].

IV. Transmission Techniques

The uplink channel, as characterized in Section II, is modeled as a deterministic signal attenuation dependent on the observed satellite's position. The presence of thermal noise in the receiver front end is accounted for in the uplink channel model by adding to the attenuated transmitted signal a white Gaussian noise term with a double sided power spectral density of $N_o/2$. The performance of a maximum likelihood detector in such a channel is dependent on the ratio of the received energy-per-bit, E_b, with N_o. As N_o is fixed, and receiver dependent, in order to maintain a specified bit error rate the received energy-per-bit must exceed a corresponding level. Using (1) the received signal power can be written as a function of time as:

$$P_r(t) = \frac{A^2}{2}\cdot\frac{\alpha-1}{\alpha-\cos(\omega_p t)}, \tag{3}$$

where it is assumed the antenna aperture and gain compensate for the path loss at $400km$. Even if this is not the case, since these are fixed quantities, the only effect will be to add a constant amount of additional transmit power to all the techniques and will therefore not affect the comparisons between them. In addition we denote the transmit power, P_{rf}, as $A^2/2$ for notational convenience.

By the fixing or varying of the transmit power, $A^2/2$, and the data rate, $1/T_s$, over the operating window, a specified E_b level can be met to assure a corresponding performance level. How these different methods each affect the energy used by the transmit power amplifier is examined, and this knowledge is used to determine a multiple access technique which can best exploit an energy savings.

A. Fixed-Power Fixed-Rate

By fixing the transmit power to an amount which meets the required received energy-per-bit at the worst case path attenuation, the received energy-per-bit is guaranteed to exceed that level throughout the window for a fixed data rate. Specifically, let the total number of bits to be transmitted over the window of duration T be denoted as V. The data rate, $1/T_s$, is constant throughout the window and is equal to V/T. To meet a specified bit error rate for a particular receiver, a received energy-per-bit of E_{b_o} is required. From (3) the worst path attenuation occurs at $\pm T/2$; the received energy-per-bit, E_b, at that time is

$$E_b(T/2) = \frac{A^2}{2}T_s\cdot\frac{\alpha-1}{\alpha-\cos(\omega_p T/2)}.$$

Setting this level to E_{b_o}, substituting for T_s, and solving for the transmit power we have

$$P_{rf} = \frac{A^2}{2} = \frac{E_{b_o}V}{T}\cdot\frac{\alpha-\cos(\omega_p T/2)}{\alpha-1}, \tag{4}$$

guaranteeing that $E_b(t) \geq E_{b_o}$ for $t \in \left[-\frac{T}{2}, \frac{T}{2}\right]$.

To generate a transmit signal of this power a class B power amplifier operating just below the compression point ($\gamma = 1$) will require a power $P_{pa} = P_{rf}\cdot\frac{4}{\pi}$. The energy-per-bit for this scheme used by the surface unit's power amplifier is therefore $E_{b_{pa}} = P_{rf}\cdot\frac{T}{V}\cdot\frac{4}{\pi}$. Substituting in for the transmit power, (4), gives

$$E_{b_{pa}} = E_{b_o}\cdot\frac{\alpha-\cos(\omega_p T/2)}{\alpha-1}\cdot\frac{4}{\pi}. \tag{5}$$

B. Variable-Power Fixed-Rate

By varying the transmit power inversely proportionally to the path attenuation over the transmission window a constant received energy-per-bit can be maintained. Let the data rate, $1/T_s$, be fixed to V/T, where V is the number of bits and T is the transmission window duration. Setting the transmit power to vary with the inverse of the path attenuation, (1), gives

$$P_{rf}(t) = \frac{A^2}{2}\cdot\frac{\alpha-\cos(\omega_p t)}{\alpha-1}. \tag{6}$$

Accounting for the path attenuation, (1), the received energy-per-bit is constant, $E_b = \frac{A^2}{2}\frac{T}{V}$. A specified received energy-per-bit, E_{b_o}, can thus be obtained by setting $A^2/2$ to $E_{b_o}V/T$.

To generate a transmit signal with this variable power level will require a class B power amplifier to operate over the range of its output power levels and associated efficiencies. The instantaneous power used to generate the transmit power, is given as $P_{pa}(t) = P_{rf}(t)\cdot\frac{4}{\gamma\pi}$, where γ^2 is the variation in transmit power, with unity normalized to the maximum RF power,

$$\gamma = \sqrt{\frac{\alpha-\cos(\omega t)}{\alpha-\cos(\omega_p\frac{T}{2})}}.$$

Substituting in the RF transmit power (6) gives

$$P_{pa}(t) = E_{b_o}\frac{V}{T}\cdot\frac{\sqrt{\alpha-\cos(\omega_p t)}}{\alpha-1}\frac{4}{\pi}\sqrt{\alpha-\cos\left(\omega_p\frac{T}{2}\right)} \tag{7}$$

The total energy used by the surface unit power amplifier in transmitting the V bits is obtained by integrating the instantaneous power, (7), over the window $[-T/2, T/2]$; using the double angle rule, $\cos(2x) = 1-2\sin^2(x)$, and making the substitution $x = \theta/2$, this integral is recognized as an incomplete elliptic integral of the second kind, as defined in [3]; dividing this by the number of bits gives us the energy-per-bit used by the surface unit's transmit power amplifier:

$$E_{b_{pa}} = E_{b_o}\frac{16}{\omega_p T\pi}\sqrt{\frac{\alpha-\cos(\omega_p\frac{T}{2})}{\alpha-1}}E\left(\frac{\omega_p T}{4}, \frac{-2}{\alpha-1}\right) \tag{8}$$

C. Fixed-Power Variable-Rate

A specified received energy-per-bit can be obtained in a channel with a changing path attenuation by varying the data rate over the window to compensate for the changing power level. Specifically, to maintain a fixed E_b, the bit rate, T_s, can be varied proportionally to the inverse of the received power, (1), giving

$$T_s(t) = \frac{\bar{T}_s(\alpha - \cos(\omega_p t))}{\alpha - 1},$$

where $\bar{T}_s$ is the minimum bit time. The average received energy-per-bit is then

$$E_b = P_{rf} \cdot \frac{\alpha - 1}{\alpha - \cos(\omega_p t)} T_s(t) = \frac{A^2}{2} \cdot \bar{T}_s. \qquad (9)$$

The total data volume over a window, $\left[-\frac{T}{2}, \frac{T}{2}\right]$, is given by the integral of the instantaneous data rate over the transmission time; this gives

$$V = \frac{1}{\bar{T}_s} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{\alpha - 1}{\alpha - \cos(\omega_p t)} dt.$$

Using the substitution $u = \tan(\frac{\omega_p t}{2})$ and solving for the maximum symbol rate, $1/\bar{T}_s$, we have

$$\frac{1}{\bar{T}_s} = \frac{V\omega_p}{4\sqrt{\frac{\alpha-1}{\alpha+1}} \arctan\left(\sqrt{\frac{\alpha+1}{\alpha-1}} \tan\left(\frac{\omega_p T}{4}\right)\right)}. \qquad (10)$$

With this variable rate system, from (9), the relative transmit power necessary to achieve a fixed bit error rate is given by $P_{rf} = A^2/2 = E_{b_o}/\bar{T}_s$. Substituting in (10) for the maximum data rate gives

$$P_{rf} = E_{b_o} \frac{V\omega_p}{4\sqrt{\frac{\alpha-1}{\alpha+1}} \arctan\left(\sqrt{\frac{\alpha+1}{\alpha-1}} \tan\left(\frac{\omega_p T}{4}\right)\right)}.$$

To generate a transmit signal of this power, a class B amplifier operating near the compression point ($\gamma = 1$) would require a power $P_{pa} = P_{rf} \cdot 4/\pi$. The energy-per-bit used by the transmit power amplifier to send V bits over a window of duration T is obtained by multiplying this power by the window duration T and dividing by the number of bits,

$$E_{b_{pa}} = E_{b_o} \frac{\omega_p T/\pi}{\sqrt{\frac{\alpha-1}{\alpha+1}} \arctan\left(\sqrt{\frac{\alpha+1}{\alpha-1}} \tan\left(\frac{\omega_p T}{4}\right)\right)}. \qquad (11)$$

D. Comparison

For an omni-directional antenna with a minimum elevation angle of ϕ_{min} communicating with an orbiting satellite in a $400km$ circular polar orbit, we evaluate (5), (8), and (11) for the additional energy-per-bit, beyond E_{b_0}, required by the transmit power amplifier to meet or exceed this received energy-per-bit level. Fig. 2 contains a plot of these energy-per-bit savings versus the minimum elevation angle. As the minimum elevation angle increases the variation in the path loss decreases, and the methods converge to that of the fixed-power fixed-rate scheme.

The fixed-power variable-rate scheme has the lowest energy-per-bit of the three transmission schemes. By varying the data rate and maintaining a fixed transmit power it allows the transmit power amplifier to operate in its most efficient region while maintaining a fixed E_b. Energy is not wasted in the transmission signal, as in the fixed-power fixed-rate system, or in the signal generation, as in the variable-power fixed-rate scheme. Since the channel is not perfectly known, the data rate cannot be varied continuously and must therefore be changed in discrete steps. This disjoint changing of the data rate makes it incompatible with either an FDMA or TDMA multiple access system. For an FDMA system the changing data rate would require re-acquisition of the symbol timing. In addition, the changing system bandwidth would also complicate carrier and phase tracking loops as well as large channel spacings, which would reduce the overall capacity of the system. A TDMA system would similarly suffer from the reassignment of slots between users to account for the changing data rates. A CDMA system, however, does not suffer from bandwidth expansion as the data rate changes. This combined with the shape of the variation in path attenuation for each unit, over a window, points to a CDMA system with a fixed-power variable-rate scheme as having the greatest potential to minimize the transmit transceiver energy expended.

V. CDMA SYSTEM

In a CDMA system all of the users transmit their data, modulated with a unique spreading code, simultaneously in the same frequency band. The receiver then uses knowledge of the spreading sequence to extract each user's data. In order to determine how much of the transmit power amplifier energy savings, shown in Section IV for a variable rate system, can be achieved with a CDMA system, we will analyze a system employing long spreading codes and using the sub-optimal single user correllator detector receiver. This will provide a performance measure for a low complexity receiver as well as a bound on the superior performance that can be expected for the more complex multi-user detector receiver.

A. Transmit Signal

The transmitted signal for each of the K users is formed from a sequence of independent data bits uniformly chosen from the alphabet $\{-1, 1\}$ at rate R. This sequence is then spread to a rate R_c with a PN chip sequence $\{c_n\}$. The ratio of the chip rate to the data rate is the processing gain, which is denoted by N_k for the k^{th} user. This spread sequence is then passed through a chip shaping filter. A rectangular chip shaping filter, $p(t)$, is chosen since this eliminates the need for a linear transmit power amplifier response, allowing it to be operated at the edge of or in the more efficient saturation region. The signal then modulates an RF carrier, $\cos(\omega_c t)$. The transmitted signal for the k^{th} user can written in its low-pass equivalent form as $x(t) = \Re\{x_l(t)e^{j\omega_c t}\}$, where

$$x_l(t) = A \sum_{n=-\infty}^{\infty} d^{(k)}_{\lfloor \frac{n}{N_k} \rfloor} c^{(k)}_n p(t - nT_c).$$

The transmitted signals are received at the orbiting satellite, each with its own delay, attenuation, and Doppler frequency shift associated with the relative position between each transmitter and the satellite.

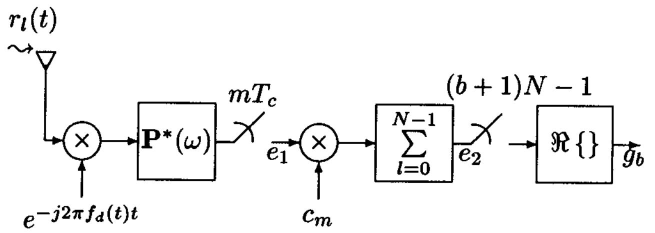

Fig. 1. Receiver

B. Received Signal

The received signal is written in a low pass equivalent form with respect to the carrier frequency ω_c, as $r(t) = \Re\{r_l(t)e^{j\omega_c t}\}$, where

$$r_l(t) = \sum_{n=-\infty}^{\infty} d^{(0)}_{\lfloor \frac{n}{N} \rfloor} c_n^{(0)} p(t-nT_c-\tau_0) g(t-\tau_0) e^{j2\pi f_d(t-\tau_o)(t-\tau_0)}$$
$$+ \sum_{k=1}^{K-1} \sum_{n=-\infty}^{\infty} d^{(k)}_{\lfloor \frac{n}{N_k} \rfloor} c_n^{(k)} p(t-nT_c-\tau_k) g(t-\tau_k) \cdot e^{j2\pi f_d(t-\tau_k)(t-\tau_k)}$$
$$+ N(t),$$

with $f_d(t)$ given by (2) and

$$g(t) = A\sqrt{\frac{\alpha-1}{\alpha-\cos(\omega_p t)}},$$

and where the low-pass equivalent noise term, $N(t)$, is a zero mean, circularly symmetric, white Gaussian process with a double sided power spectral density $2N_o$.

C. Test Statistic

Assuming, for surface unit zero, we have perfect frequency, phase, chip, and symbol synchronization, we set $\tau_0 = 0$ and use the low-pass equivalent single user correllator detector receiver structure shown in Fig. 1. After reception the Doppler shift of the 0^{th} unit is compensated for. The received signal is then passed through the chip matched filter, which is sampled at the chip time, mT_c. As the path attenuation and Doppler terms are assumed to change at a rate much less than the data rate, they are considered static over a chip time. Using this along with the small angle approximations for both sine and cosine, we can write

$$e_1(mT_c) = d^{(0)}_{\lfloor \frac{m}{N} \rfloor} c_m^{(0)} g(mT_c) T_c$$
$$+ \sum_{k=1}^{K-1} g(mT_c-\tau_k) e^{j\left[\omega_m^{(k)}(mT_c-\tau_k) - \omega_m^{(0)}(mT_c)\right]}$$
$$\times \left[d^{(k)}_{\lfloor \frac{m-l-1}{N_k} \rfloor} c^{(k)}_{m-l-1} \hat{\tau}_k + d^{(k)}_{\lfloor \frac{m-l}{N_k} \rfloor} c^{(k)}_{m-l} (T_c - \hat{\tau}_k) \right]$$
$$+ e^{-\omega_m^{(0)} mT_c} N_m.$$

where $E[N_m N_n^*] = 2N_o T_c \delta_{m,n}$, and τ_k is equal to l integer multiples of T_c with a remainder of $\hat{\tau}_k$.

These samples are then multiplied by the spreading sequence of the 0^{th} user, a sum of the previous N of these samples is formed, and this is sampled at the bit time of 0^{th} user. As both the path attenuation and Doppler terms vary at a rate much less than the symbol time; they are considered constant over the bit period and are pulled out of the sum over i. Assuming $\{c^{(k)}\}$ has a period much greater than the bit time, we can model each user's spreading sequences as independent random binary sequences. The data term can be dropped from the multiple access terms as it has no effect; this results in

$$e_2((b+1)N-1) = g(bNT_c)NT_c d_b^{(0)}$$
$$+ \sum_{k=1}^{K-1} I_k(b) + e^{-\omega_{bN}^{(0)} bNT_c} \sum_{i=0}^{N-1} N_{(b+1)N-1-i},$$

where

$$I_k(b) = g(bNT_c-\tau_k) e^{j\left[\omega_{bN}^{(k)}(bNT_c-\tau_k) - \omega_{bN}^{(0)}(bNT_c)\right]}$$
$$\cdot \sum_{i=0}^{N-1} \left[c^{(k)}_{(b+1)N-1-i-l-1} c^{(0)}_{(b+1)N-1-i} \hat{\tau}_k \right.$$
$$\left. + c^{(k)}_{(b+1)N-1-i-l} c^{(0)}_{(b+1)N-1-i} (T_c - \hat{\tau}_k) \right].$$

The multiple access term for the k^{th} user, $I_k(b)$, is the sum of independent random variables; we can therefore apply the central limit theorem and model it as an independent zero-mean complex Gaussian random variable.

Assuming $\{c_n\}$ has good autocorrelation properties, and making the substitution $\tau_k = lT_c + \hat{\tau}_k$, the conditional variance of this random variable is

$$E\left[I_k(b) I_k(b)^* | \left\{c^{(0)}\right\}, \left\{\omega_{bN}^{(0)}\right\}\right]$$
$$= E\left[A^2(bNT_c - lT_c - \hat{\tau}_k) \cdot (\hat{\tau}_k^2 N + N(T_c - \hat{\tau}_k)^2)\right].$$

As previously stated, the path attenuation does not vary over the chip time; we therefore drop $\hat{\tau}_k$ from its argument. We further assume that the path delay of other surface units is uniformly distributed over $[-T/2 + bNT_c, T/2 + bNT_c]$. This in essence assumes the other users are uniformly distributed over the operating window; we therefore write the variance as $E[A^2(\gamma)]\, E[\hat{\tau}_k^2 N + N(T_c - \hat{\tau}_k)^2]$, where γ is independent of $\hat{\tau}_k$ and is uniformly distributed in $[-T/2, T/2]$. The expectation with regard to $\hat{\tau}_k$ and γ can be evaluated giving,

$$E\left[I_k(b) I_k(b)^* \middle| \left\{c^{(0)}\right\}, \left\{\omega_{bN}^{(0)}\right\}\right]$$
$$= \frac{2T_c^2}{3} \frac{4A^2}{\omega_p T} arctan\left(\frac{\alpha+1}{\alpha-1} \tan\left(\frac{\omega_p T}{4}\right)\right).$$

We are also interested in showing that the multiple access terms are circularly symmetric. Using the same assumptions as above this can be shown by demonstrating that the variance of the real and imaginary parts of $I_k(b)$ are equal. The sum of these multiple access terms is therefore a complex circularly symmetric Gaussian random variable.

The test statistic, g_b, for the b^{th} bit of the 0^{th} user is formed by taking the real part of $e_2(((b+1)N-1)T_c)$, which gives

$$g_b = A(bNT_c)NT_c d_b^{(0)} + \Re\{I + \nu\},$$

where

$$I = \sum_{k=1}^{K-1} I_k(b) \quad \text{and} \quad \nu = e^{-\omega_{bN}^{(0)} bNT_c} \sum_{i=0}^{N-1} N_{(b+1)N-1-i}.$$

Both I and ν are independent, zero-mean, circularly symmetric Gaussian random variables. The real part of their sum is a real Gaussian random with a variance of

$$E\left[\Re\{I+\nu\}\Re\{I+\nu\}\right] = \frac{4(K-1)A^2T_c^2}{3}\frac{arctan\left(\frac{\alpha+1}{\alpha-1}\tan\left(\frac{\omega_p T}{4}\right)\right)}{\omega_p T} + NN_oT_c.$$

D. Probability of Bit Error

Having characterized the test statistic, we can now solve for the probability of bit error. Given that each symbol is independently and uniformly chosen, and since the channel is symmetric, the average probability of bit error can then be written as $P_e = Pr(g < 0|d_b = 1)$ and evaluated in terms of the distribution function of $I+\nu$.

In an ideal system the data rate would change proportionally to the inverse of the received signal path attenuation power, maintaining the same E_b/N_o over the entire window. In our system this results in a changing processing gain

$$N = \frac{\bar{T}}{T_c}\frac{\alpha - \cos(\omega_p NbT_c)}{\alpha - 1}.$$

Using this and the energy-per-bit expended by the transmit power amplifier, (11), the probability of bit error can be written as

$$P_e = \phi\left(-\left[\frac{4(K-1)}{3\bar{N}}\frac{arctan\left(\frac{\alpha+1}{\alpha-1}\tan\left(\frac{\omega_p T}{4}\right)\right)}{\omega_p T} + \frac{1}{\frac{2E_{bpa}}{N_o}\frac{4}{\omega_p T}\sqrt{\frac{\alpha-1}{\alpha+1}}\arctan\left(\sqrt{\frac{\alpha+1}{\alpha-1}}\tan\left(\frac{\omega_p T}{4}\right)\right)\cdot\frac{\pi}{4}}\right]^{-\frac{1}{2}}\right) \quad (12)$$

where $\bar{N} = \bar{T}/T_c$ and ϕ is the cumulative density function of a zero mean unit variance Gaussian random variable.

To compare this to an orthogonal multiple access scheme, such as FDMA or TDMA, using variable-power fixed-rate or fixed-power fixed-rate, we note the probability of bit error in terms of the received energy-per-bit for these systems is written as $\phi(-\sqrt{2E_b/N_o})$. Substituting the received energy-per-bit with the equivalent transmit power amplifier energy-per-bit for the fixed-power fixed-rate, (5), and variable-power fixed-rate, (8) one obtains the probability of bit error.

Fig. 3, for a minimum elevation angle of 15 degrees, contains a plot of the performance of the fixed-power variable-rate CDMA system, (12), for various number of user, K, to processing gain, $\bar{N}$, ratios, along with those of the fixed-power fixed-rate and variable-power fixed rate orthogonal multiple access schemes. Also included in these plots is a curve representing an ideal orthogonal fixed-power variable-rate system; with a number of users to minimum processing gain ratio of $1/16$ the CDMA system comes within a fraction of a dB of this ideal performance. As a single user detector was used in the analysis, the processing gain linearly mitigates the effects of the multiple access interference. The use of a multi-user detector would provide a greater reduction in MAI, allowing the CDMA system performance to more closely achieve that of the ideal fixed-power variable-rate system.

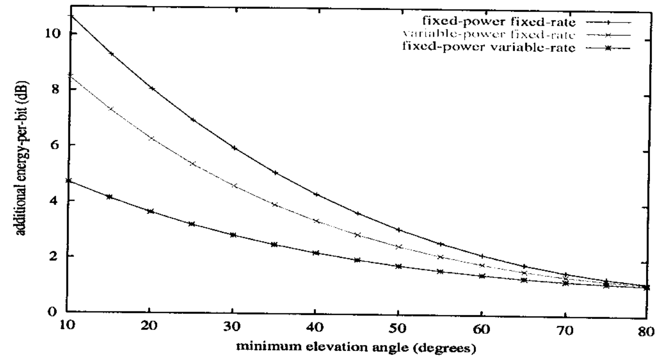

Fig. 2. The additional transmit energy-per-bit required to meet or exceed a specified received energy-per-bit for different minimum elevation angle scenarios.

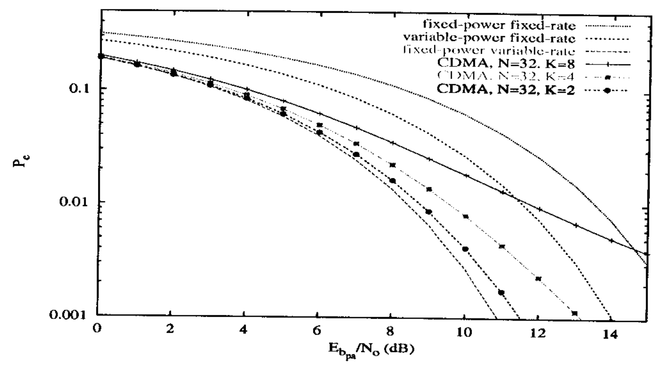

Fig. 3. Probability of bit error versus the energy-per-bit at the power amplifier for a minimum elevation angle of 15 degrees.

VI. Conclusion

The minimization of the transmit power amplifier energy-per-bit required to meet a specified probability of bit error was used as the performance measure for choosing a multiple access technique which will provide the longest battery life for a Martian surface unit transceiver. Using this criterion several different transmission schemes for the uplink channel were considered, and a fixed-power variable-rate transmission scheme was determined to provide the greatest energy savings. Due to a unique property of spread spectrum signaling, namely no bandwidth expansion for changing data rates, a CDMA system was chosen as being best able to take advantage of the energy savings in fixed-power variable-rate signaling. The performance of a single user detector CDMA system was derived and comparisons were made between this and fixed-power fixed-rate, variable-power fixed rate orthogonal multiple access systems (FDMA, TDMA). The potential savings were shown to be on the order several dB, which would significantly extend the battery life of a Martian surface unit transceiver.

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